Self-Insurance in Turbulent Labor Markets

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Supplementary Appendix: Not for Publication

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A Data Appendix

A.1 Data Description

For the empirical analysis, we use data from the NLSY79, a nationally longitudinal survey of 12,696 individuals who were between 14 and 22 years old when they were first interviewed in 1979. We use a cross-sectional sample comprising 6,111 respondents, designed to represent the non-institutionalized civilian population living in the United States in 1979, with ages ranging from 14 to 22 as of December 31, 1978. From the core sample, we exclude individuals who served in the military for more than two years because their military experience is not representative of the standard civilian expertise. We further exclude respondents who are weakly attached to the labor market, defined as those who have been out of a job for more than 10 years.

Worker's employment history The NLSY79 interviewed individuals annually from 1979 to 1993 and biannually from 1994 to 2016. Information on labor force status is recorded weekly throughout the sample period, even in the later period when interviews were conducted biannually. Following Baley, Figueiredo and Ulbricht (2022), we use the NLSY79's Work History Data file to construct a monthly panel. This file is a week-by-week record of each respondent's working history, including weekly labor status and hours worked. While an individual may hold more than one job, we focus on the primary job in a given month, defined as the one for which the individual worked the most hours. Using a mapping that links jobs across consecutive interviews, we build a panel report of employment spells for the primary job and any individuals who have not worked.

For each primary job, we retain information on the hourly wage, occupation, and industry codes. Before merging occupation and industry information with the employment panel, we clean occupational and industry titles using the approach outlined in Guvenen, Kuruscu, Tanaka and Wiczer (2020). For each job, we assign the occupation and industry codes most often observed during the employment spell. In the NLSY79, occupation titles are described by the three-digit Census occupation code. Because this classification system changed over time[26], before cleaning we converted all the occupational codes across the years into the *occ1990dd* occupation system developed by Dorn (2009), which has the advantage of being time-consistent.[27]

Following Kambourov and Manovskii (2008), we mitigate measurement error by assigning each job the modal occupation code for its spell and classifying a switch as genuine only when it coincides with an employer change. Using this definition, we find that about 52% of EUE transitions involve an occupation switch, in line with the 45–67% range reported by Huckfeldt (2022) for the CPS Displaced Worker Supplement. Among EUE transitions without an occupational switch, around

[26] Until 2000, NLSY79 reports occupation codes in the Census 1970 three-digit occupation code. After this year, occupation codes are reported in the Census 2000 three-digit occupation code.

[27] The crosswalk files between the Census classification codes and the *occ1990dd* occupation aggregates created by Autor and Dorn (2013) can be found at `http://www.ddorn.net/data`.

35% are recalls to a previous employer, close to the 40% share reported by Fujita and Moscarini (2017) using the SIPP. These results confirm that even under a more traditional coding-based approach, our measurement strategy delivers credible benchmarks and a consistent mapping to the model's distinction between turbulent transitions (involving skill depreciation) and tranquil ones.

Assets The NLSY79 contains detailed questions on household asset holdings and liabilities from 1985 onwards. The wealth information is not observed at the same frequency as the labor market data; asset data are collected at interview dates, yielding at most one observation per year. The NLSY79 defines all assets as the amount the respondent would reasonably expect someone to pay if the asset were sold in its current condition. Respondents report the market value of their assets at the time of the interview; this information is thus assigned to its particular calendar month, leaving all others blank. We use the CPI reported by the BLS to convert each asset's market value to 2000 dollars. From the detailed information reported by NLSY, we create five categories of net assets—residential property, financial assets, business assets, vehicles, and others—as follows:

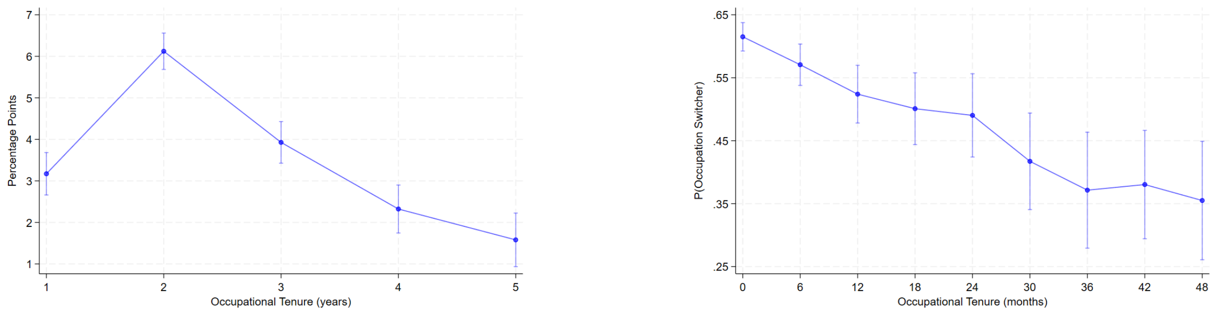
- Residential Property = "Market value of residential property r/spouse own" - "Amount of mortgages and back taxes r/spouse owe on residential property"
- Financial assets = "Total market value of stocks/bonds/mutual funds" + "Total amount of money assets like savings accounts of r/spouse"+ "Total amount of money in assets like IRAS or KEOUGH of r/spouse"+"Total amount of money in assets like CDS, loans or mortgages of r/spouse"'
- Business assets = "Total market value of farm/business/other property r/spouse own"-"Total amount of debts on farm/business/other property r/spouse owe"
- Vehicles = "Total market value of vehicles including automobiles r/spouse own"- "Total amount of money r/spouse owe on vehicles including automobiles"
- Others = "Total market value of all other assets each worth more than $500"-"Total amount of other debts over $500 r/spouse owe"

We then define *Liquid Wealth* as the sum of business assets, financial assets, vehicles, and other assets, all net of debts. Following Lise (2013), we trim the top and bottom one-half of one percent of the distribution of the assets to reduce the influence of outliers.

A.2 Turbulence Risk

Figure A.1 illustrates why we distinguish between untenured and tenured workers in defining turbulence risk. Panel (a) shows that wage returns to occupational tenure are concentrated in the first two years, consistent with the rapid acquisition of occupation-specific knowledge. Panel (b)

Figure A.1: Occupational Tenure and Mobility



(a) Returns to Occupational Tenure (b) Occupational Mobility

Note: Panel (a) plots the wage return to working an additional year in an occupation during the first four years of employment in that occupation. Estimates control for individual characteristics (age and its square, labor market experience, gender, race, educational attainment, and ability), year, and month fixed effects. Panel (b) plots the probability of switching occupation from a logit regression that includes individual characteristics (age and its square, labor market experience, gender, race, educational attainment, and previous hourly wage), as well as year and month fixed effects. Both panels use EUE transitions observed from 1979 to 2016 in the NLSY79.

shows that the probability of switching occupations declines sharply during the first 30 months and stabilizes thereafter. These patterns justify classifying workers with less than two years of tenure as untenured, while labeling subsequent occupational switches as potential turbulent transitions involving skill loss.

A.3 Summary Statistics

The following table provides summaries of EUE transitions and worker characteristics at separation:

Table A.1: Summary Statistics

	All	**Untenured**	**Tranquil**	**Turbulent**
EUE transitions	37,324	25,910	7,102	4,212
% of total	100	69.4	19.0	11.6
Worker characteristics at separation				
Age (years)	29.7	26.8	36.6	36.0
Job tenure (years)	1.4	0.5	3.0	3.6
Occupational tenure (years)	2.5	0.7	7.2	5.8
Labor market experience (years)	8.3	5.7	14.8	13.5
Hourly wage (2000 dollars)	11.5	9.5	17.3	13.5
Liquid wealth (000's, 2000 dollars)	28.9	20.1	43.0	35.2
Outcomes at reemployment				
Wage growth Δw (in %)	1%	4%	0%	-12%

Notes: Average characteristics of EUE transitions observed from 1979 to 2016 in NLSY79.

A.4 Robustness Checks

In this section, we show that the empirical findings are robust to:

(i) unemployment benefits and spousal income as controls (Figure A.2),

(ii) different thresholds for occupational tenure τ (Figure A.3),

(iii) excluding short unemployment spells (Figure A.4),

(iv) restricting the sample to EUE transitions of workers with high job tenure in the previous job (at least two years) to better isolate involuntary separations from quits and selective firings (Figure A.5),

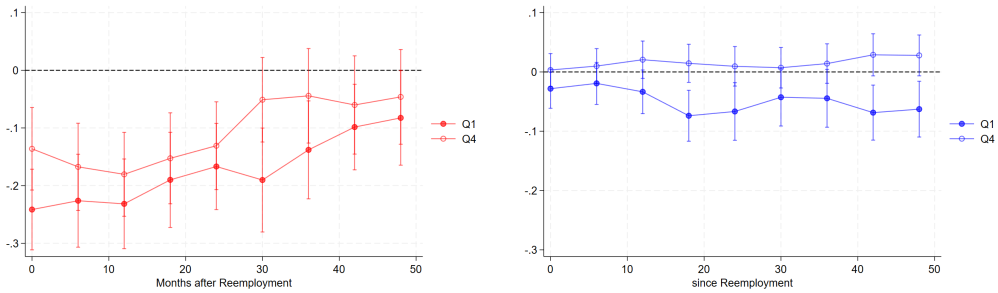
(v) different definitions of liquid wealth (Figure A.6),

(vi) a more traditional definition of turbulence-based occupational mobility across 3-digit occupation codes, in contrast to our baseline turbulence measure based on angular distances in the space of occupational skills requirements.

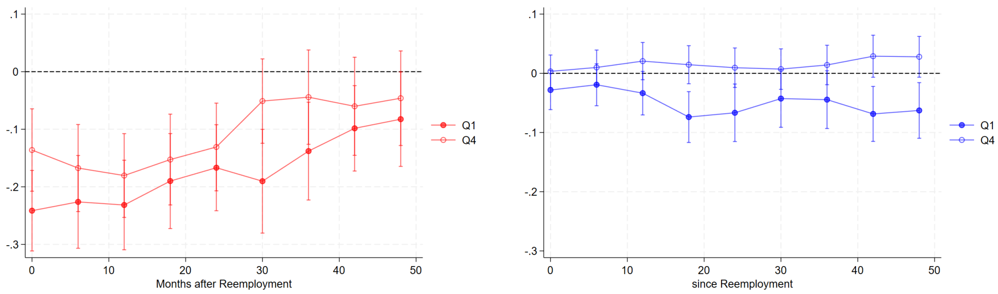
In all figures, we plot average residual wage growth at reemployment and every six months thereafter. The left plots shows turbulent transitions, defined as workers who switch to a new occupation with low skill transferability, i.e., those experiencing a turbulence shock following an unemployment spell. The right plots show tranquil transitions, defined as workers who, after unemployment, either return to their previous occupation or move to one with high skill transferability. Q1 and Q4 denote the first and fourth quartiles of the household liquid wealth distribution at the start of the unemployment spell. The sample covers EUE' transitions observed from 1979 to 2016 in NLSY79.

These robustness checks suggest that, regardless of how we approach the data, we find similar-sized gaps between high- and low-liquid-wealth workers across turbulent and tranquil transitions. Our empirical findings reveal an essential feature of the wage cost of job loss. There is significant ex-post heterogeneity in wage outcomes among unemployed workers. For workers who do not experience a turbulence shock, the wage cost of job loss does not appear particularly severe. However, wage costs are significant and persistent for workers with low liquidity who experience In contrast, workers who experience a turbulence shock, but have high liquid wealth upon job loss, can recover from the immediate wage loss within four years.

Figure A.2: Robustness Check: Additional Controls

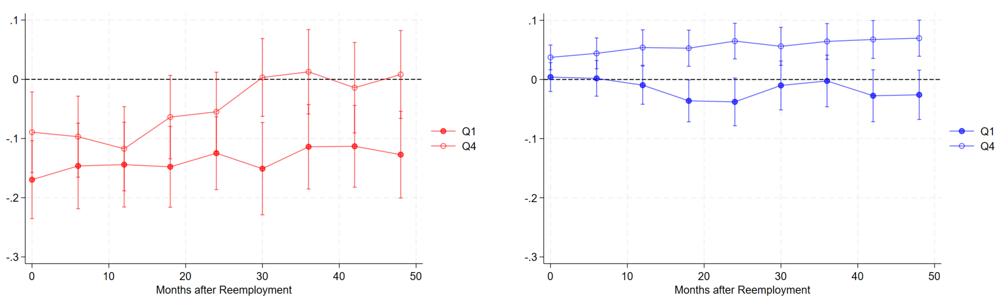


(a) Spousal Income

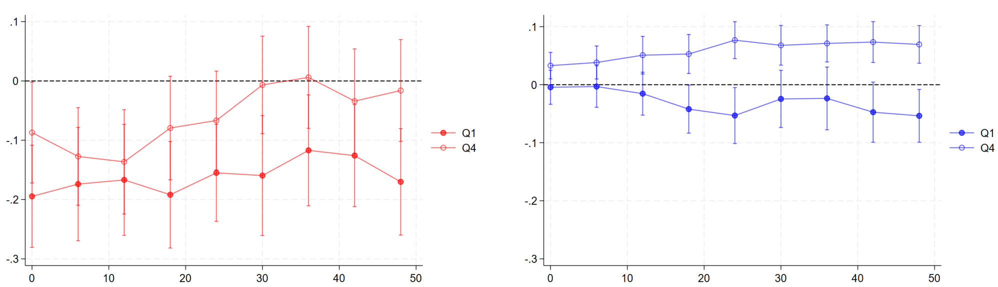


(b) Unemployment Benefits

Figure A.3: Robustness Check: Alternative Occupational Tenure Threshold (τ)

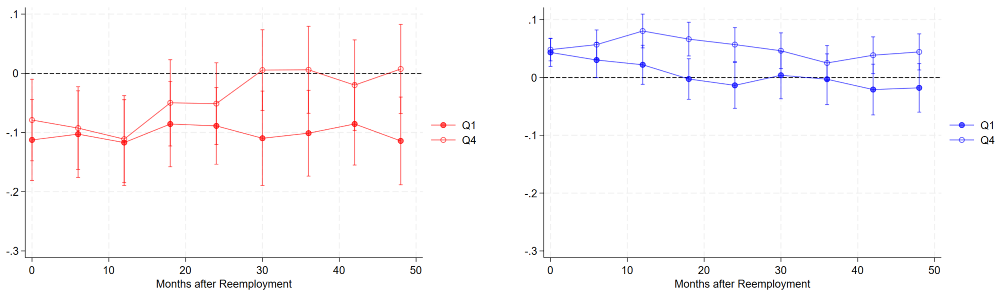


(a) Tenure Threshold Set at One Year

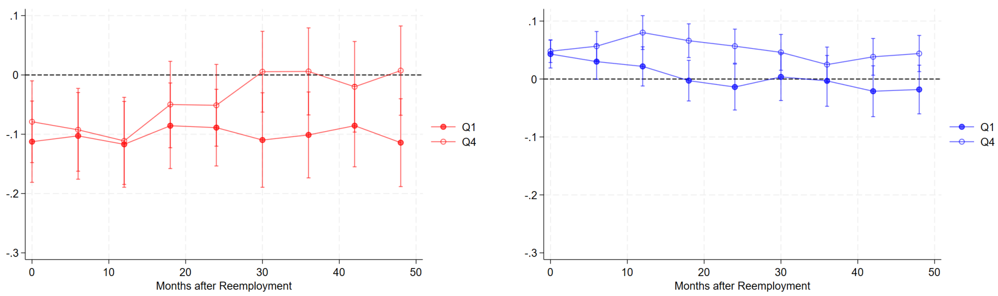


(b) Tenure Threshold Set at Three Years

Figure A.4: Robustness Check: Excluding Short Unemployment Spells



(a) Excluding Spells Lasting 1 Month



(b) Excluding Spells Lasting Less Than 2 Months

Figure A.5: Robustness Check: Restrict to High Job Tenure Workers

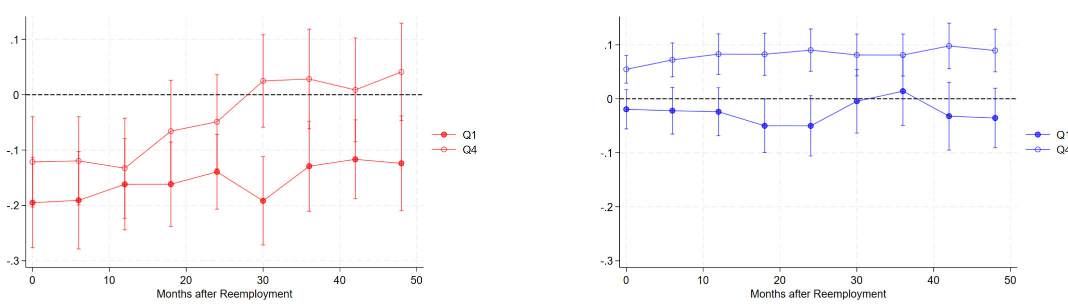
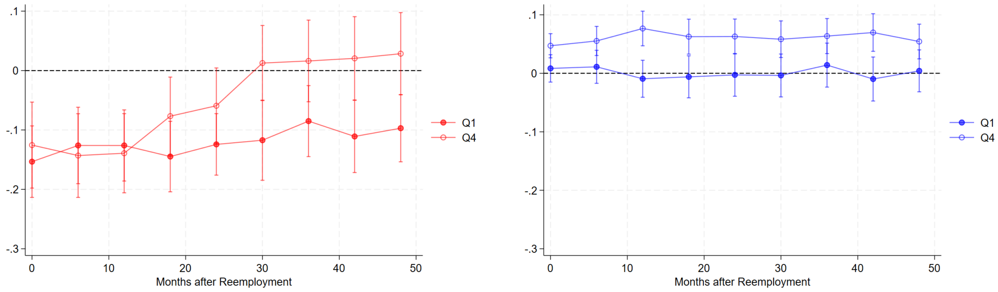
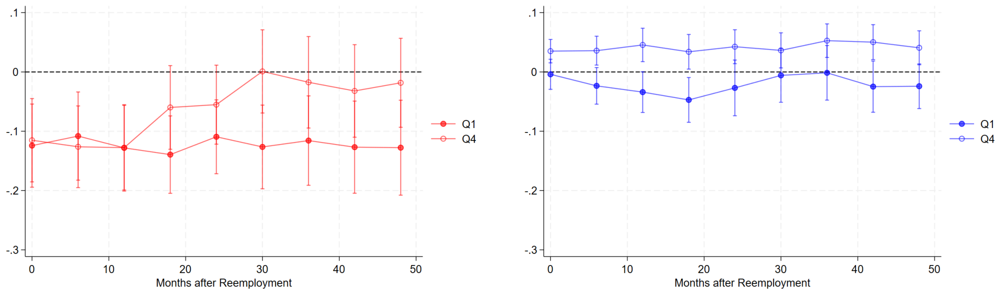


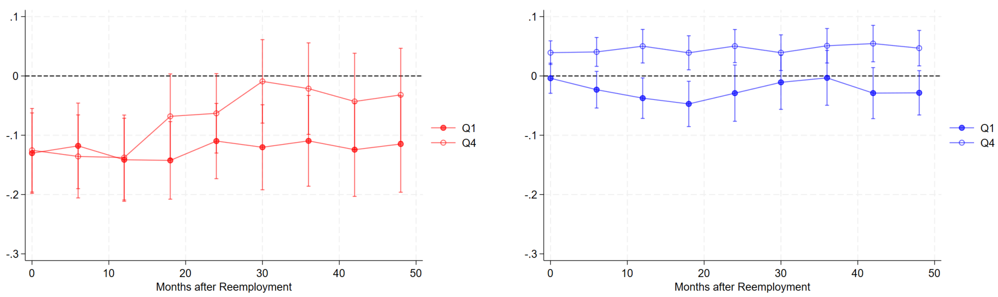
Figure A.6: Robustness Check: Different Definitions of Liquid Wealth



(a) L1

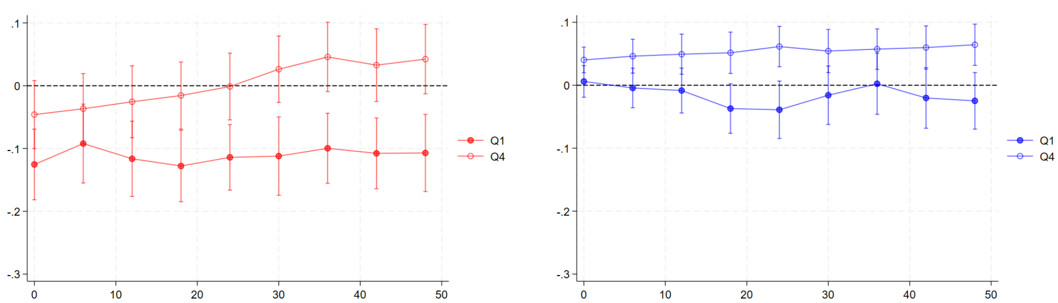


(b) L2



(c) L3

Figure A.7: Robustness Check: Switching Across 3-digit Occupations



B Proofs and Derivations

B.1 Solution to the firm problem.

We now detail the steps to solve the firm problem that gives rise to a wage-tightness menu. We proceed in two steps. First, we derive a relationship between values of filled jobs and tightness on one hand, and wages on the other hand. Then, in the second step, we put these conditions together to derive a wage-tightness menu for workers to choose from.

Values of filled jobs. Free entry of firms implies that the steady-state value of posting a vacancy in all submarkets in (4) is equal to zero $V_{ik} = 0$ for all skill i and tier k. This implies that since all firms are identical, they are indifferent to hiring workers with different assets-skills within each tier. This is because of the trade-off between the probability of filling the vacancy and the wages firms offer. From $V_{ik} = 0$, we obtain the following zero profit condition:

$$J_{ik} = \frac{\kappa_k}{\beta q_k(\theta)}, \qquad i \in \{l, k\}. \tag{B.1}$$

Condition (B.1) represents a relationship between job values and tightness; we now derive a relationship between job values and wages. To do so, set $V_{hk} = 0$ in the value function (5). This yields

$$J_{hk}(w) = f_{hk} - w + \beta\left[(1-\lambda_{hk})J_{hk}(w)\right] \tag{B.2}$$

Solving for $J_{hk}(w)$ yields:

$$J_{hk}(w) = \frac{f_{hk} - w}{1-\beta(1-\lambda_{hk})} \tag{B.3}$$

Setting $V_{lk} = 0$ in the value function (6) yields

$$J_{lk}(w) = f_{lk} - w + \beta(1-\lambda_{lk})\left((1-\gamma^u)J_{lk}(w) + \gamma^u J_{hk}(w')\right) \tag{B.4}$$

Substituting $J_{hk}(w')$ using (B.3) evaluated at the upgraded wage yields:

$$J_{lk}(w) = f_{lk} - w + \beta(1-\lambda_{lk})\left((1-\gamma^u)J_{lk}(w) + \gamma^u\left(\frac{f_{hk} - w'}{1-\beta(1-\lambda_{hk})}\right)\right) \tag{B.5}$$

Joining $J_{lk}(w)$ on the LHS delivers

$$J_{lk}(w)[1-\beta(1-\lambda_{lk})(1-\gamma^u)] = f_{lk} - w + \beta(1-\lambda_{lk})\gamma^u\left(\frac{f_{hk} - w'}{1-\beta(1-\lambda_{hk})}\right) \tag{B.6}$$

Next, in the first line, we substitute the upgraded wage $w' = w + (f_{hk} - f_{lk})$; in the second line, we define $\widehat{\gamma_k^u} \equiv \frac{\beta(1-\lambda_{lk})\gamma^u}{1-\beta(1-\lambda_{hk})}$; in the third line, we solve for $J_{lk}(w)$:

$$\begin{aligned} J_{lk}(w)[1-\beta(1-\lambda_{lk})(1-\gamma^u)] &= f_{lk} - w + \beta(1-\lambda_{lk})\gamma^u \left(\frac{f_{hk} - w - (f_{hk} - f_{lk})}{1-\beta(1-\lambda_{hk})}\right) \\ J_{lk}(w)[1-\beta(1-\lambda_{lk})(1-\gamma^u)] &= f_{lk} - w + \widehat{\gamma_k^u}(f_{lk} - w) \\ J_{lk}(w) &= \frac{f_{lk} - w + \widehat{\gamma_k^u}[f_{lk} - w]}{[1-\beta(1-\lambda_{lk})(1-\gamma^u)]} \end{aligned} \tag{B.7}$$

Summarizing, job values can be expressed as function of wages (and model parameters) as:

$$J_{hk}(w) = \frac{f_{hk} - w}{1-\beta(1-\lambda_{hk})}, \tag{B.8}$$

$$J_{lk}(w) = \frac{f_{lk}(1+\widehat{\gamma_k^u}) - w(1+\widehat{\gamma_k^u})}{[1-\beta(1-\lambda_{lk})(1-\gamma^u)]} \tag{B.9}$$

where the adjusted upgraded probability is $\widehat{\gamma_k^u} \equiv \frac{\beta(1-\lambda_{lk})}{1-\beta(1-\lambda_{hk})}\gamma^u$.

Wage-tightness menu. For high-skilled workers, we substitute the free entry condition $J_{hk} = \kappa_k/(\beta q_k(\theta))$ in (B.1) into the value of filled jobs with high-skilled workers in (B.3):

$$\frac{f_{hk} - w}{1-\beta(1-\lambda_{hk})} = \frac{\kappa_k}{\beta q_k(\theta)} \tag{B.10}$$

Solving for the wage:

$$w_{hk}(\theta) = f_{hk} - \frac{\kappa_k(1-\beta(1-\lambda_{hk}))}{\beta q_k(\theta)} \tag{B.11}$$

Finally, defining $\tilde{\kappa}_{hk} \equiv \kappa_k(1-\beta(1-\lambda_{hk}))/\beta$ yields the menu for high-skilled workers in tier k:

$$w_{hk}(\theta) = f_{hk} - \frac{\tilde{\kappa}_{hk}}{q_k(\theta)}. \tag{B.12}$$

For low-skilled workers, we substituting the free entry condition $J_{lk} = \kappa_k/(\beta q_k(\theta))$ in (B.1) into the value of filled jobs with low-skilled workers in (B.7) yields:

$$\frac{f_{lk} - w + \widehat{\gamma_k^u}(f_{lk} - w)}{(1-\beta(1-\lambda_{lk})(1-\gamma^u)} = \frac{\kappa_k}{\beta q_k(\theta)} \tag{B.13}$$

Solving for the wage

$$w_{lk}(\theta) \;=\; \frac{f_{lk} + \widehat{\gamma_k^u}(f_{lk}\;)}{1+\widehat{\gamma_k^u}} \;-\; \frac{\kappa_k((1-\beta(1-\lambda_{lk})(1-\gamma^u))}{(1+\widehat{\gamma_k^u})\beta q_k(\theta)} \tag{B.14}$$

Finally, defining $\tilde{\kappa}_{lk} \equiv \kappa_k(1-\beta(1-\lambda_{lk})(1-\gamma^u)/\beta$ yields the menu for low-skilled workers in tier k:

$$w_{lk}(\theta) \;=\; \frac{f_{lk} + \widehat{\gamma_k^u}(f_{lk}\;)}{1+\widehat{\gamma_k^u}} \;-\; \frac{\tilde{\kappa}_{lk}}{(1+\widehat{\gamma_k^u})q_k(\theta)}. \tag{B.15}$$

Summarizing, we have derived an equilibrium relationship between tightness and wages that can be summarized by the equations:

$$w_{hk}(\theta) \;=\; f_{hk} - \frac{\tilde{\kappa}_{hk}}{q_{hk}(\theta)}, \tag{B.16}$$

$$w_{lk}(\theta) \;=\; \frac{f_{lk}(1+\widehat{\gamma_k^u})}{1+\widehat{\gamma_k^u}} \;-\; \frac{\tilde{\kappa}_{lk}}{(1+\widehat{\gamma_k^u})q_k(\theta)} \tag{B.17}$$

Finally, note that in the previous expressions, $\tilde{\kappa}_{hk}$ and $\tilde{\kappa}_{lk}$ can be interpreted as the effective hiring costs in the respective tier-skill submarket.

Firms are indifferent between paying high wages and hiring fast or paying low wages but hiring slow. The figure also highlights that the wage-tightness menu available to workers is tier-skill specific. The following section explains that workers optimally choose the submarket where they apply for jobs, given their skills and assets.

B.2 Workers' optimality conditions

Assets. The FOC for assets (a') are given by (with slight abuse of notation, we denote ν as the multiplier here)

$$\begin{aligned}
\frac{\partial u(c_{ij}^u)}{\partial c} &= \beta R\left[m(\theta)\frac{\partial u(c_i^{e\prime})}{\partial c} + (1-m(\theta))\frac{\partial u(c_{ij}^{u\prime})}{\partial c}\right] + \nu^{u_{ij}} \\
\frac{\partial u(c_l^e)}{\partial c} &= \beta R\left\{\lambda\frac{\partial u(c_l^{u\prime})}{\partial c} + (1-\lambda)\left[\gamma^u\frac{\partial u(c_h^{e\prime})}{\partial c} + (1-\gamma^u)\frac{\partial u(c_l^{e\prime})}{\partial c}\right]\right\} + \nu^{e_l} \\
\frac{\partial u(c_h^e)}{\partial c} &= \beta R\left\{\lambda\left[\gamma^d\frac{\partial u(c_{lh}^u)}{\partial c} + (1-\gamma^d)\frac{\partial u(c_{hh}^u)}{\partial c}\right] + (1-\lambda)\frac{\partial u(c_h^e)}{\partial c}\right\} + \nu^{e_h}
\end{aligned}$$

where $\nu^{x_{ij}}$ measures the shadow value of the borrowing constraint if it is binding.

Proof. The FOC for an unemployed worker with type i, j is given by:

$$\frac{\partial u(c_{ij}^u)}{\partial c} = \beta\left\{m(\theta)\frac{\partial E_i}{\partial a'} + (1-m(\theta))\frac{\partial U_{ij}}{\partial a'}\right\} + \nu^{u_{ij}} \tag{B.18}$$

The envelope condition for U is $\frac{\partial U_{ij}}{\partial a} = R\frac{\partial u(c^u_{ij})}{\partial a} + \beta\left[\frac{\partial m(\theta)}{\partial\theta}\frac{\partial\theta}{\partial a}(E_i - U_{ij})\right] = R\frac{\partial u(c^u_{ij})}{\partial a}$, since by optimality, $\frac{\partial\theta}{\partial a} = 0$, and the envelope condition for E_i is $\frac{\partial E_i}{\partial a} = R\frac{\partial u(c^e_i)}{\partial a}$. Substituting the envelope conditions, evaluated at $t+1$, we obtain the standard Euler equation:

(B.19)
$$\frac{\partial u(c^u_{ij})}{\partial c} = \beta R\left[m(\theta)\frac{\partial u(c^{e\prime}_i)}{\partial c} + (1 - m(\theta))\frac{\partial u(c^{u\prime}_{ij})}{\partial c}\right] + \nu^{u_{ij}}$$

where $\nu > 0$ measures the shadow value of the borrowing constraint when it is binding.

For a low-skilled employed worker the FOC reads:

(B.20)
$$\frac{\partial u(c^e_l)}{\partial c} = \beta\left\{\lambda\frac{\partial U_l}{\partial a'} + (1-\lambda)\left[\gamma^u\frac{\partial E_h}{\partial a'} + (1-\gamma^u)\frac{\partial E_l}{\partial a'}\right]\right\} + \nu^{e_l}$$

Substituting using the envelope conditions:

(B.21)
$$\frac{\partial u(c^e_l)}{\partial c} = \beta R\left\{\lambda\frac{\partial u(c^{u\prime}_l)}{\partial c} + (1-\lambda)\left[\gamma^u\frac{\partial u(c^{e\prime}_h)}{\partial c} + (1-\gamma^u)\frac{\partial u(c^{e\prime}_l)}{\partial c}\right]\right\} + \nu^{e_l}$$

Finally, the FOC for a high skilled employed worker reads:

(B.22)
$$\frac{\partial u(c^e_h)}{\partial c} = \beta\left\{\lambda\left[\gamma^d\frac{\partial U_{lh}}{\partial a'} + (1-\gamma^d)\frac{\partial U_{hh}}{\partial a'}\right] + (1-\lambda)\frac{\partial E_h}{\partial a'}\right\} + \nu^{e_h}$$

Substituting the envelope conditions:

(B.23)
$$\frac{\partial u(c^e_h)}{\partial c} = \beta R\left\{\lambda\left[\gamma^d\frac{\partial u(c^{u\prime}_{lh})}{\partial c} + (1-\gamma^d)\frac{\partial u(c^{u\prime}_{hh})}{\partial c}\right] + (1-\lambda)\frac{\partial u(c^{e\prime}_h)}{\partial c}\right\} + \nu^{e_h}$$

□

Submarket choice. Within a given tier, FOC for tightness (θ) is characterized by the following equation:

$$\underbrace{\mathcal{E}_{m,\theta}}_{\text{increase in finding prob}} \quad \underbrace{\frac{E-U}{E}}_{\text{gains from leaving unemp.}} \quad = \quad \underbrace{-\mathcal{E}_{w,\theta}}_{\text{wages lost}} \quad \underbrace{\mathcal{E}_{E,w}}_{\text{sensitivity of utility to wages}}$$

Proof. The FOC of the workers' problem with respect to tightness θ is given by: [28]

$$\begin{aligned} m'(\theta)\left(E-U\right)+m(\theta)\frac{\partial E}{\partial w}\frac{\partial w}{\partial \theta} &= 0 \\ \frac{m'(\theta)}{m(\theta)}\left(E-U\right)+\frac{\partial E}{\partial w}\frac{\partial w}{\partial \theta} &= 0 \\ \frac{m'(\theta)}{m(\theta)}\theta\left(\frac{E-U}{E}\right)+\frac{\partial E}{\partial w}\frac{\partial w}{\partial \theta}\frac{\theta}{w}\frac{w}{E} &= 0 \end{aligned}$$

Using the definition of elasticities, we can rewrite the last condition as:

$$\mathcal{E}_{m,\theta}\left(\frac{E-U}{E}\right) = -\mathcal{E}_{w,\theta}\mathcal{E}_{E,w} \tag{B.24}$$

where $\mathcal{E}_{m,\theta} \equiv \frac{m'(\theta)}{m(\theta)}\theta$ denotes the elasticity of the finding probability to tightness, $\mathcal{E}_{w,\theta} \equiv \frac{\partial w}{\partial \theta}\frac{\theta}{w}$ the elasticity of the wage profile to tightness, and $\mathcal{E}_{E,w} \equiv \frac{\partial E}{\partial w}\frac{w}{E}$ the elasticity of the utility of being employed to the wage. Condition (B.24) has a very intuitive interpretation: at an optimum, the marginal benefit of choosing a submarket with higher tightness (given by the left hand side) has to be equal to its marginal cost; the marginal benefit is represented by a higher job-finding probability times the gain from becoming employed; the marginal cost is captured by how much wages vary with tightness ($\mathcal{E}_{w,\theta}$) times how much wages influence the value of being employed, given by $\mathcal{E}_{E,w}$. $\square$

B.3 Unemployed Value Function $\mathcal{U}$: Derivation

To derive equation (10), first note that if two independent random variables ϵ_1, ϵ_2 with distributions $G_1(x)$ and $G_2(x)$, the distribution of $max(\epsilon_1, \epsilon_2)$ is simply the product of the individual distributions, i.e. $G_1 \cdot G_2$.

We want to find the distribution of the maximum of the k' variables $U_{k'}(a - M_{kk'}, x_{ij}) + \nu\epsilon_{k'}$. Given the assumption of Type I Extreme Value shocks, define

$$Z = \max_{k'}\Big(U_{k'}(a - M_{kk'}, x_{ij}) + \nu\epsilon_{k'}\Big) \tag{B.25}$$

[28]This applies for every skill and tier considered, so to save on notation, we omit subscripts and superscripts and we show the derivation for given value functions U and E.

and denote its cdf by $M(x) \equiv \Pr(Z \leq x)$. Then, for every possible value of k_j, $j = 1, 2, 3, \ldots$:[29]

$$M(x) = \exp\Big(-\exp\Big(-\frac{x - U_{k_1}(a - M_{kk_1}, x_{ij})}{\nu}\Big)\Big) \cdot \exp\Big(-\exp\Big(-\frac{x - U_{k_2}(a - M_{kk_2}, x_{ij})}{\nu}\Big)\Big) \\ \cdot \exp\Big(-\exp\Big(-\frac{x - U_{k_3}(a - M_{kk_3}, x_{ij})}{\nu}\Big)\Big) \cdots \\ = \exp\Big(-\exp\Big(-\frac{x - \omega}{\nu}\Big)\Big), \tag{B.26}$$

where $\omega = \nu \log \sum_{k'} \exp\big(\frac{1}{\nu} \cdot U_{k'}(a - M_{kk'}, x_{ij})\big)$. This is equivalent to say that Z is a random variable that is distributed according to a Type I Extreme Value distribution, with location parameter ω and scale ν; hence $\mathbb{E}[Z] = \omega + \gamma\nu$, so (up to the additive constant $\gamma\nu$) this proves the result.

B.4 Switching Probability: Derivation

In this section, we derive the expression for the endogenous probability of switching tier, as expressed in (11).

Under the assumption that shocks are Type I - Extreme Value distributed, the probability that a worker of skill x_{ij} in tier k chooses to move to tier k', defined by $\mu_{kk'}(a, x_{ij})$, is:

$$\mu_{kk'}(a, x_{ij}) = \Pr\big(U^x_{k'} + \nu\epsilon^i_{k'} > U^x_r + \nu\epsilon^i_r,\ \forall r \neq k'\big) \tag{B.27}$$

$$= \Pr\big((U^x_{k'} - U^x_r) + \nu\epsilon^i_{k'} > \nu\epsilon^i_r,\ \forall r \neq k'\big) \tag{B.28}$$

$$= \Pr\big(\tfrac{1}{\nu}(U^x_{k'} - U^x_r) + \epsilon^i_{k'} > \epsilon^i_r,\ \forall r \neq k'\big), \tag{B.29}$$

where $U^x_r = U(a - M_{kr}, x, r)$. The last expression is simply the cdf of ϵ^i_r evaluated at $\frac{1}{\nu}(U^x_{k'} - U^x_r) + \epsilon^i_{k'}$. Since ϵ^i_r is iid across r, this cdf can be computed easily as

$$\mu_{kk'}(a, x_{ij}) = \int \Bigg[\prod_{r \neq k'} \exp\big(-\exp\big(-\big(\tfrac{1}{\nu}(U^x_{k'} - U^x_r) + \epsilon\big)\big)\big)\Bigg] \exp(-\epsilon - \exp(-\epsilon))\ d\epsilon \tag{B.30}$$

where the integral is over $\epsilon \equiv \epsilon^i_{k'}$. Calculating this integral gives the expression in the main text:

$$\mu_{kk'}(a, x_{ij}) = \frac{\exp\big(\frac{1}{\nu}U(a - M_{kk'}, x, k')\big)}{\sum_{r \in \{A,B\}} \exp\big(\frac{1}{\nu}U(a - M_{kr}, x, r)\big)}. \tag{B.31}$$

[29]Recall the cdf $F(x)$ of the Type I Extreme Value distribution is $F(x) = \exp\big(-\exp\big(-\frac{x-\mu}{\nu}\big)\big)$ where μ and ν are the location and scale parameters, respectively.

B.5 Profits

For each (i, k), net profits are zero:

$$\pi_{ik}^{net} \;=\; \pi_{ik}^{gross} - \kappa_k v_{ik} \tag{B.32}$$

where gross profits are

$$\pi_{ik}^{gross} \;=\; \int (f_{ik} - w)\; d\Gamma^{e_{ik}}(w) \tag{B.33}$$

and total vacancies are inferred from the vacancy filling rates evaluated at optimal wage policies:

$$v_{ik} = \int u_{ik}(a)\theta_{ik}^*(a) d\Gamma^{u_{ik}}(a) \quad \text{and} \quad \theta_{ik}^*(a) = \left[\left(\frac{q(w_{ik}^*(a))}{\chi_{ik}}\right)^{-\alpha} - 1\right]^{1/\alpha}. \tag{B.34}$$

Average gross profits of active firms equal the production minus the wage:

$$\pi^{gross} \;=\; \sum_{k,i}(f_{ik} - w_{ik}), \quad \text{where} \quad w_{ik} = \int w\; d\Gamma^{e_{ik}}(w). \tag{B.35}$$

The total vacancy creation cost
(B.36)

$$\kappa \;=\; \sum_{k,i} \kappa_k v_{ik}, \quad \text{where} \quad v_{ik} = \int u_{ik}(a)\theta_{ik}^*(a) d\Gamma^{u_{ik}}(a) \quad \text{and} \quad \theta_{ik}^*(a) = \left[\left(\frac{q(w_{ik}^*(a))}{\chi_{ik}}\right)^{-\alpha} - 1\right]^{1/\alpha}$$

Then, net profits (taking into account) are equal to:

$$\pi^{net} = \pi^{g} - \kappa \tag{B.37}$$

B.6 Welfare measure

We begin by defining our notion of welfare comparisons across steady-states, which are indexed with different government policies. Consider the economy in a steady state. At date $t = 0$, a policy ϕ is introduced. Consider two consumption sequences, the "baseline" $\{c_t\}_{t=0}^{\infty}$ and the "alternative" $\{\tilde{c}_t\}_{t=0}^{\infty}$, where the second considers a different policy. At the individual level, our welfare measure quantifies the consumption change required to deliver our counterfactual worker the same lifetime utility in the alternative as in the baseline.

Individual welfare For a worker with skill $i \in \{l, h\}$, labor status $s \in \{E, U\}$ in tier $k \in \{A, B\}$, assets a_0 and wage w_0 (or b if unemployed), the lifetime utility is given by:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{((1 + \lambda_{ik}^s(a_0, w_0)|\phi)c_t)^{1-\sigma}}{1-\sigma} \right] = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{\tilde{c}_t^{1-\sigma}}{1-\sigma} \right]. \tag{B.38}$$

Solving for $\lambda_{ik}^s(a_0, w_0)$ yields:

$$\lambda_{ik}^s(a_0, w_0|\phi) \;=\; \left(\frac{\mathbb{E}_0 \Big[\sum_{t=0}^{\infty} \beta^t (\tilde{c}_t)^{1-\sigma} \Big]}{\mathbb{E}_0 \Big[\sum_{t=0}^{\infty} \beta^t (c_t)^{1-\sigma} \Big]} \right)^{\frac{1}{1-\sigma}} - 1. \tag{B.39}$$

If $\lambda > 0$, the worker is better off in the alternative scenario.

Aggregate welfare To aggregate the individual welfare measure across groups of workers—defined by skill, tier, or assets—we average individual welfare using the distributions of unemployed $\Gamma^{u_{ik}}(a)$ and employed $\Gamma^{e_{ik}}(a, w)$, which satisfy

$$\sum_{i \in \{l,h\}} \sum_{k \in \{A,B\}} \left[\int \Gamma^{u_{ik}}(a) da + \int \int \Gamma^{e_{ik}}(a, w) da dw \right] = 1. \tag{B.40}$$

Welfare of unemployed The welfare measures at various aggregation levels are:

$$\text{(individual)} \qquad \lambda_{ik}^u(a_0|\phi) = \left(\frac{\mathcal{U}_{ik}(a_0)}{\mathcal{U}_{ik}(a_0|\phi)} \right)^{\frac{1}{1-\sigma}} - 1 \tag{B.41}$$

$$\text{(over skills/tiers)} \qquad \lambda_{ik}^u(\phi) \equiv \int \lambda_{ik}^u(a_0|\phi) \Gamma^{u_{ik}}(a_0) da_0 \tag{B.42}$$

$$\text{(aggregate)} \qquad \lambda^u(\phi) = \sum_{ik} \lambda_{ik}^u(\phi) \Gamma^{u_{ik}} \tag{B.43}$$

where $\Gamma^{u_{ik}} = \int \Gamma^{u_{ik}}(a_0) da_0$ is the mass of unemployed workers with skill i in tier k.

Welfare of employed The welfare measures at various aggregation levels are:

$$\text{(B.44)} \qquad \text{(individual)} \qquad \lambda^e_{ik}(a_0, w_0|\phi) = \left(\frac{E_{ik}(a_0, w_0)}{E_{ik}(a_0, w_0|\phi)}\right)^{\frac{1}{1-\sigma}} - 1$$

$$\text{(B.45)} \qquad \text{(over skills/tiers)} \qquad \lambda^e_{ik}(\phi) = \int \left[\int \lambda^e_{ik}(a_0, w_0|\phi)\ \Gamma^{e_{ik}}(a_0, w_0) dw_0\right] da_0$$

$$\text{(B.46)} \qquad \text{(over assets)} \qquad \lambda^e_{ik}(a_0|\phi) = \int \lambda^e_{ik}(a_0, w_0|\phi)\ \Gamma^{e_{ik}}(w_0, a_0) dw_0,$$

$$\text{(B.47)} \qquad \text{(over wages)} \qquad \lambda^e_{ik}(w_0|\phi) = \int \lambda^e_{ik}(a_0, w_0|\phi)\ \Gamma^{e_{ik}}(w_0, a_0) da_0,$$

$$\text{(B.48)} \qquad \text{(aggregate)} \qquad \lambda^e(\phi) = \sum_{ik} \lambda^e_{ik}(\phi) \Gamma^{e_{ik}}.$$

where $\Gamma^{e_{ik}} = \int \int \Gamma^{e_{ik}}(a_0, w_0) da_0 dw_0$ is the mass of employed workers with skill i in tier k.

Economy-wide welfare Finally, the economy-wide consumption equivalent:

$$\text{(B.49)} \qquad \lambda(\phi) = \lambda^e(\phi)\Gamma^e + \lambda^u(\phi)\Gamma^u$$

where $\Gamma^e = \sum_{i,k} \Gamma^{e_{ik}}$ and $\Gamma^u = \sum_{i,k} \Gamma^{u_{ik}}$ are the total number of employed and unemployed.

C Computational Appendix

We describe the computation details of the algorithm used to solve and estimate the model. The model is block-recursive, and hence the policy functions for assets, job search, and tier choice can be solved separately from the steady state distributions over assets and wages. We now illustrate how we compute these equilibrium objects (policies and distributions).[30]

C.1 General structure

The algorithm to solve the model starts by solving the firms' problem, which, as illustrated in the mathematical appendix, delivers an endogenous wage-tightness menu from which workers can choose. This step is particularly fast, and it represents the first step in the algorithm. Next, we need to solve for the workers' problems: choosing assets and, if unemployed, choosing tier and submarket. We exploit the Euler equation to reduce the computational time required to compute the model's equilibrium. Given job finding and tier switching probabilities $m(\theta)$ and $\mu_{kk'}$, the solution for consumption can be found by iteratively looking for the consumption decision that is consistent with the Euler equation given by the standard consumption-savings problem. The additional difficulty is that this requires doing so for a given $m(\theta)$ and $\mu_{kk'}$, which are endogenous and hence require an additional step in which we solve for them.[31] We now illustrate the technical details used in the implementation of the algorithm.

Stage 1. Find the wage-tightness schedule.

1. For a given tier k, discretize wage space for high-skill unemployed workers in the space $w(x_{hk}) \in [b(x_h), y_k x_h]$.[32] For each wage in the interval, we use the zero profit condition due to free entry to find the market tightness:

$$\theta(x_{hk}) = q^{-1}\left[\frac{\kappa_h}{y_k x_h - w(x_{hk})}\right], \qquad \text{where} \qquad \tilde{\kappa}_{hk} \equiv \kappa_k(1-\beta(1-\lambda_{hk}))/\beta.$$

The CES matching function implies a vacancy filling rate of $q(\theta) = \chi(1+\theta^\alpha)^{\frac{-1}{\alpha}}$ and thus an inverse vacancy filling rate of $q^{-1}(y) = \left(\left(\frac{\chi}{y}\right)^\alpha - 1\right)^{\frac{1}{\alpha}}$, thus we have

$$\theta(x_{hk}) = \left(\left[\frac{y_k x_h - w(x_{hk})}{\kappa_{hk}/\chi}\right]^\alpha - 1\right)^{\frac{1}{\alpha}}$$

[30] When conducting the welfare exercise, we consider a tax loop to close the government budget at every steady state and for every UI or JC level. The solution to the model is then embedded in a tax loop, that searches for the tax rate that closes the budget given the policy in place.

[31] This challenge can be avoided with a deterministic life-cycle structure, as in Giannone, Li, Paixao and Pang (2023).

[32] Because the implied wage-tightness menu from the firms' condition will be non-linear, we use a denser grid where non-linearity is stronger.

Finally, the job finding probability as a function of wages is given by

$$m(w(x_{hk})) = \underbrace{\left(\left[\frac{y_k x_h - w(x_{hk})}{\kappa_{hk}/\chi}\right]^{\alpha} - 1\right)^{\frac{1}{\alpha}}}_{\theta} \underbrace{\left[\frac{\kappa_{hk}}{y_k x_h - w(x_{hk})}\right]}_{q(\theta)}$$

Then the wage-finding rate profile in the high-skilled market is given by $[w(x_{hk}), m(w(x_{hk}))]$.

2. Similarly for low-skilled in tier k, discretise wage space for low-skilled unemployed workers in the space $w(x_{lk}) \in [b(x_{lk}), y_k x_l]$. For each wage, find the market tightness using:

$$\theta(x_{lk}) = q^{-1}\left[\frac{\kappa_{lk}}{y_k x_l + \hat{\gamma^u} x_h - (1+\hat{\gamma^u})w(x_{lk})}\right]$$

where $\hat{\gamma^u} \equiv \frac{\beta(1-\lambda)\gamma^u}{1-\beta(1-\lambda)}$ and $\kappa_l \equiv \frac{(1-\beta(1-\lambda)(1-\gamma^u))\kappa}{\beta}$. The wage-finding rate profile is given by

$$[w(x_{lk}), m(\theta(x_{lk}))]$$

Stage 2. Policies and distributions

1. **Stage 2a. Given a value for τ, solve the consumption-savings problem and the labor market search problem (tier and submarket).**

   (a) Discretise asset space $[\underline{a}, a_{max}]$.

   (b) For each asset a, wage w, skill level x_j, and tier k, guess value functions for employed $E^0(a, w, x_j, k)$ and unemployed $U^0(a, w, x_j, k)$, job finding and wage policies $[w(\theta), m(\theta)]$, and tier search μ_{kk}.

   (c) *Outer Loop: Job and Tier Search* Given the guesses, solve for the tier and submarket choice, i.e., solve for $m(\theta)$ and μ_{kk} for each skill x_l, x_h.

   (d) *Inner Loop: Consumption and Savings* Given the solutions for tier and submarket choice, we use the endogenous grid point method to solve for consumptions and assets efficiently.

   (e) Iterating on the outer and inner loop, we obtain solutions for the value functions, the tier switching, the asset savings, and job market search policies.

2. **Stage 2b. Steady state distribution**

   (a) Using policy functions obtained in 2a, simulate the model to find the stationary asset distributions at the steady state for employed $\Gamma^E(a, x_i, k)$ and unemployed workers $\Gamma^U(a, x_j, k)$. See C.2 below for details on this step.

3. **Stage 2c. Clear government budget constraint**

   (a) Use the distributions to compute the mass of unemployed workers with benefit entitlement j and also the revenue that enters the government's budget constraint:

   $$ub = \tau' \int_a \int_{x_i} \sum_k w(a, x_i, k) d\Gamma^E(a, x_j, k) \tag{C.50}$$

   where

   $$u \equiv \int_a \int_{x_i} \sum_k d\Gamma_U(a, x_j, k)$$

   (b) Find the tax rate τ' that clears the government's budget.

   (c) If the new tax rate τ' is close to the previous τ, i.e. $|\tau' - \tau| < tol$, stop.

   (d) Otherwise, with the new tax rate τ', go back to the beginning of Stage 2 until convergence.

C.2 Stationary Distributions

Recalling that $i \in \{l, h\}$ denotes skill and $k \in \{A, B\}$ denotes the job tier, within each tier we track a finite set of worker types X that collect employment status and skill history. X can be one of the following five types: $X \in \{e_{ll}, e_{hh}, u_{ll}, u_{lh}, u_{hh}\}$.

For each tier-skill pair (i, k) we are interested in stationary distributions of employed workers over assets and wages, $\Gamma^E_{ik}(a, w)$, and of unemployed workers over assets, $\Gamma^U_{ik}(a)$, as defined in the main text. Stacking all these objects across i and k yields an aggregate distribution Γ over the discretized state space (a, w, X, k).

Denote the one-period transition probability by

$$\Pr(a', w, X', k' \mid a, w, X, k).$$

We can factor the transition into the asset evolution and the discrete (X, k) component as follows:

$$\Pr(a', w, X', k' \mid a, w, X, k) = \Pr(a' \mid a, w, X, k) \; \Pr(X', k' \mid a, X, k). \tag{C.51}$$

Within-tier type transitions. Fix a tier k. Conditional on not retiring or switching tier, the probability of changing type X depends on separation rates $\lambda_{lk}, \lambda_{hk}$, upgrade and downgrade probabilities γ^u, γ^d_k, and the job finding rates $m^k_{ll}(a), m^k_{lh}(a), m^k_{hh}(a)$ implied by workers' search choices in that tier. Retirement at rate ρ_r is modeled by removing a fraction ρ_r of workers from every state and reintroducing them as low–skill, low–entitlement unemployed in each tier. The transition probabilities within tier k are:

$$\Pr(X' \mid a, X, k) = \rho^r \times \begin{array}{c c} & \begin{array}{ccccc} e_{ll} & e_{hh} & u_{ll} & u_{lh} & u_{hh} \end{array} \\ \begin{array}{c} e_{ll} \\ e_{hh} \\ u_{ll} \\ u_{lh} \\ u_{hh} \end{array} & \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

$$+ (1-\rho^r) \times \begin{array}{c c} & \begin{array}{ccccc} e_{ll} & e_{hh} & u_{ll} & u_{lh} & u_{hh} \end{array} \\ \begin{array}{c} e_{ll} \\ e_{hh} \\ u_{ll} \\ u_{lh} \\ u_{hh} \end{array} & \begin{pmatrix} (1-\lambda_{lk})(1-\gamma^u) & (1-\lambda_{lk})\gamma^u & \lambda_{lk} & 0 & 0 \\ 0 & 1-\lambda_{hk} & 0 & \lambda_{hk}\gamma_k^d & \lambda_{hk}(1-\gamma_k^d) \\ m_{ll}^k(a) & 0 & 1-m_{ll}^k(a) & 0 & 0 \\ m_{lh}^k(a) & 0 & 0 & 1-m_{lh}^k(a) & 0 \\ 0 & m_{hh}^k(a) & 0 & 0 & 1-m_{hh}^k(a) \end{pmatrix} \end{array}.$$

Between-tier transitions. Tier changes occur only for unemployed workers. Recall that $\mu_{ikk'}(a)$ denotes the probability that an unemployed worker with skill i and assets a, who is currently in tier k, chooses to search in tier $k' \in \{A, B\}$ next period. These switching probabilities solve the outer tier choice and imply that the tier-skill unemployment distributions satisfy

$$\Gamma_{ik'}^U(a) = \sum_{k \in \{A,B\}} \mu_{ikk'}(a)\, \Gamma_{ik}^U(a), \qquad i \in \{l, h\},\ k' \in \{A, B\}. \tag{C.52}$$

so that the unemployed mass is reallocated across tiers according to $\mu_{ikk'}(a)$.

Combining the within-tier transition matrix above with the mobility mapping (C.52) yields a global transition operator P_T over the full state space (a, w, X, k). Stacking the distributions for both tiers into a single vector Γ, a stationary distribution is a fixed point of this operator:

$$\Gamma = P_T \Gamma, \qquad \sum_s \Gamma(s) = 1.$$

We compute Γ numerically by iterating on this law of motion from an initial guess. The tier-skill distributions $\Gamma_{ik}^E(a, w)$ and $\Gamma_{ik}^U(a)$ defined in the main text are then obtained as the relevant marginals of Γ over (X, k).

C.3 SMM Estimation Details

To perform the SMM estimation routine, we conduct a global search over the parameter space to find the vector of parameter values, $\hat{\Theta}$, that minimizes the distance between data moments ($Moments_d$) and model moments ($Moments_m$). That is, $\hat{\Theta}$ solves

$$\hat{\Theta} = \arg\min_{\theta}(Moments_d - Moments_m)'\mathcal{W}(Moments_d - Moments_m)$$

where $\mathcal{W}$ is a weigthing matrix. The global search is conducted by evaluating the objective function just described over a large set of parameter combinations indexed by a Sobol sequence, to cover the parameter space uniformly.

D Additional model outcomes

D.1 Wage and Asset Distributions

We plot the equilibrium wage and asset distributions of the model.

Figure D.1: Wage distributions (conditional)

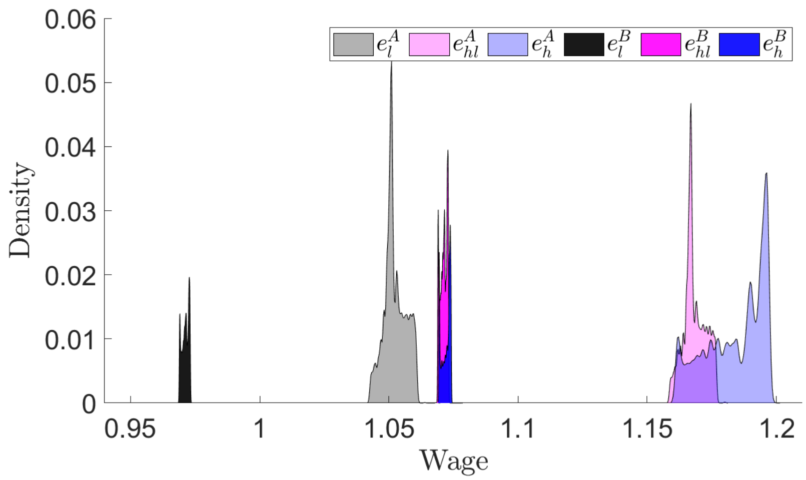
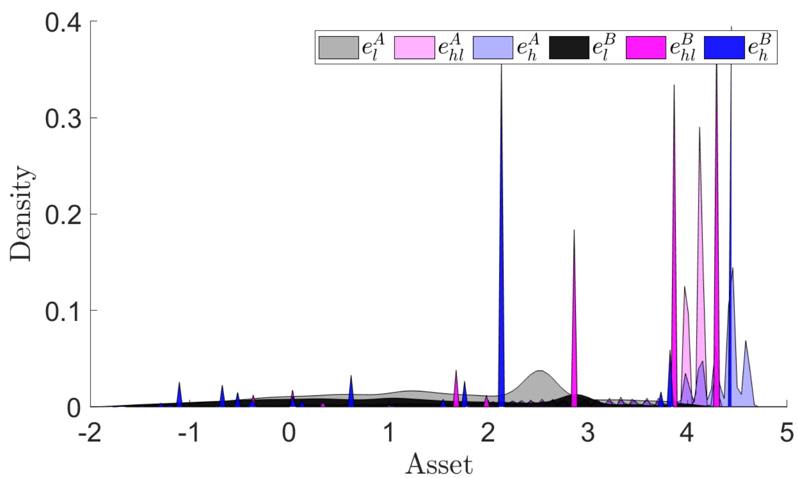


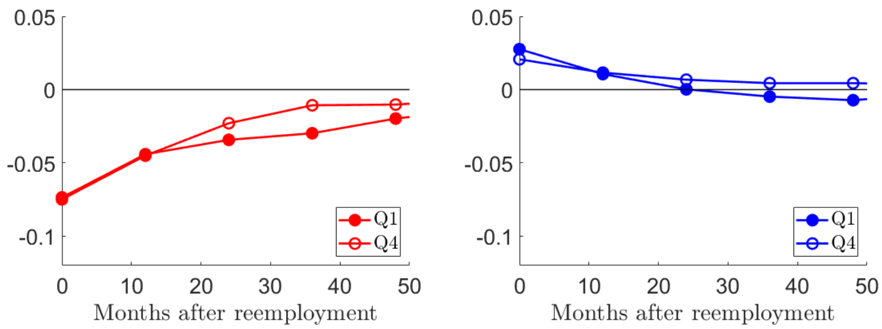
Figure D.2: Asset distributions (conditional)



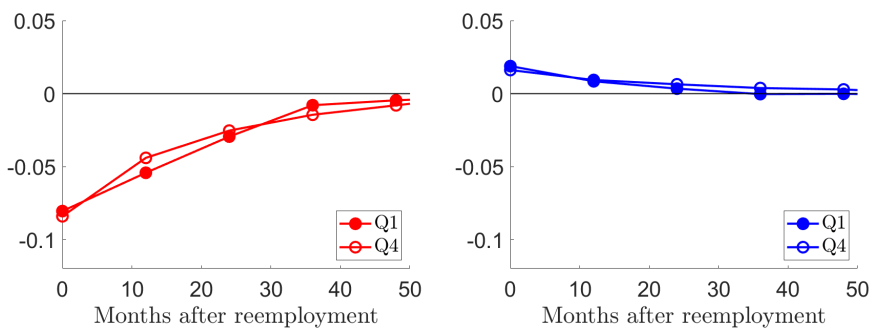
D.2 Counterfactuals

Figure D.3 shows average residual wage growth at reemployment and at six-month intervals thereafter in the model using alternative calibrations. In Panel (a), we set $\nu \to \infty$ and Panel (b) we set all parameters in Tier B equal to their counterpart in Tier A. In each panel, the left plot corresponds to turbulent transitions and the right plot to tranquil transitions. Q1 and Q4 denote the first and fourth quartiles of the household liquid wealth distribution at the start of the unemployment spell.

Figure D.3: Effects of Removing Precautionary Tier Mobility



(a) No role for wealth ($\nu \to \infty$)

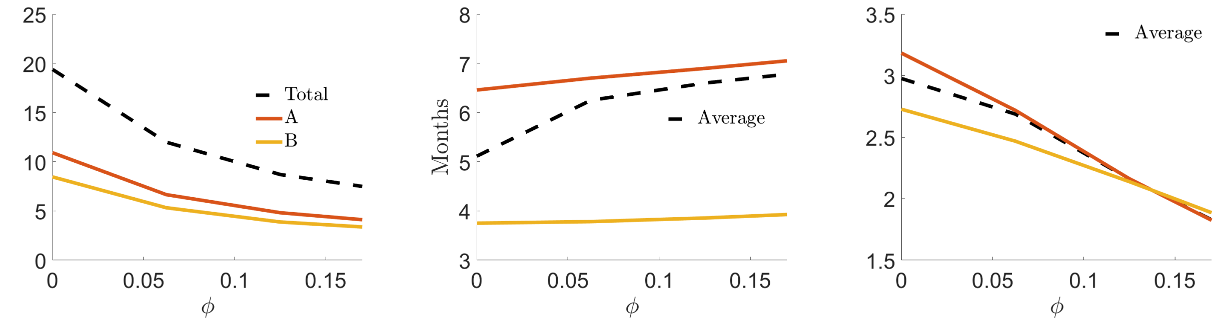


(b) One-tier Model

D.3 Precautionary Channels Across Tiers

The following figures show tier mobility rates, unemployment duration, and average assets across various policy levels. Panel (a) shows aggregate mobility rates and by tier, where the 'A' line denotes switchers to A, and the 'B' line shows switchers to B. Panel (b) shows unemployment duration in months, on average, and by tier. Finally, Panel (c) shows assets, on average, and by tier. All outcomes are equilibrium values under varying UI generosity ϕ or job creation subsidy ω.

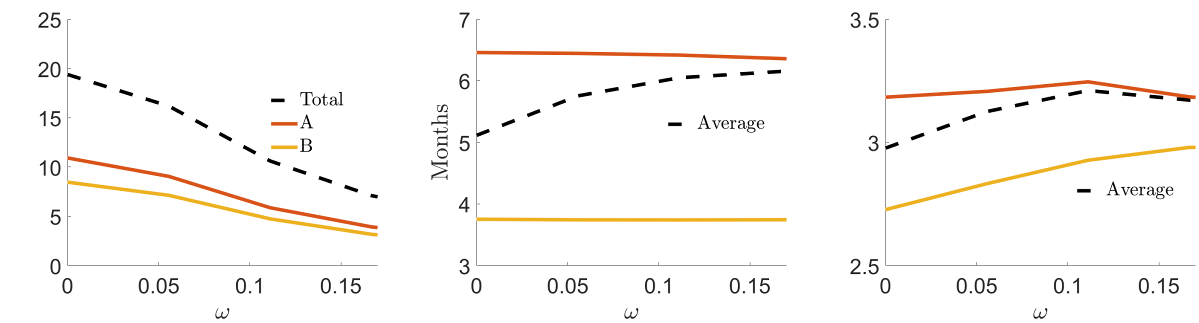
Figure D.4: Precautionary channels for different unemployment benefits ϕ



(a) Tier Mobility Rates (b) Unempl. duration (c) Average assets

Note: Panel (a) shows tier mobility rates (aggregate and by tier; 'A' line denotes switchers to A, and 'B' line shows switchers to B); Panel (b) shows unemployment duration in months, on average, and by tier; and Panel (c) shows assets, on average, and by tier. All outcomes are equilibrium values under varying UI generosity ϕ.

Figure D.5: Precautionary channels by tier for job creation subsidy ω



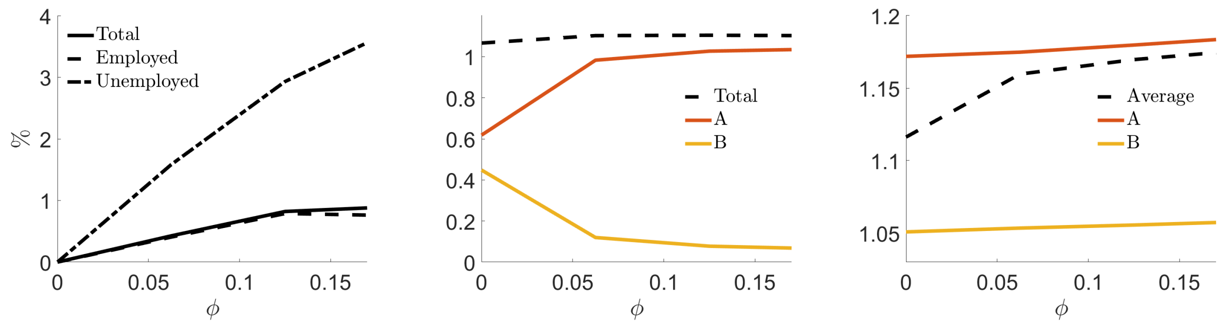
(a) Tier Mobility Rates (b) Unempl. duration (c) Average assets

Note: Panel (a) shows tier mobility rates (aggregate and by tier; 'A' line denotes switchers to A, and 'B' line shows switchers to B); Panel (b) shows unemployment duration in months, on average and by tier; and Panel (c) shows assets, on average and by tier. All outcomes are equilibrium values under varying job creation subsidy ω.

D.4 Macro Outcomes Across Tiers

The following figures show welfare, output, and wages across various policy levels. Panel (a) shows welfare, aggregate and by employment status; Panel (b) shows output, total and by tier; and Panel (c) shows wages, average and by tier. All outcomes are equilibrium values under varying UI generosity ϕ or job creation subsidy ω.

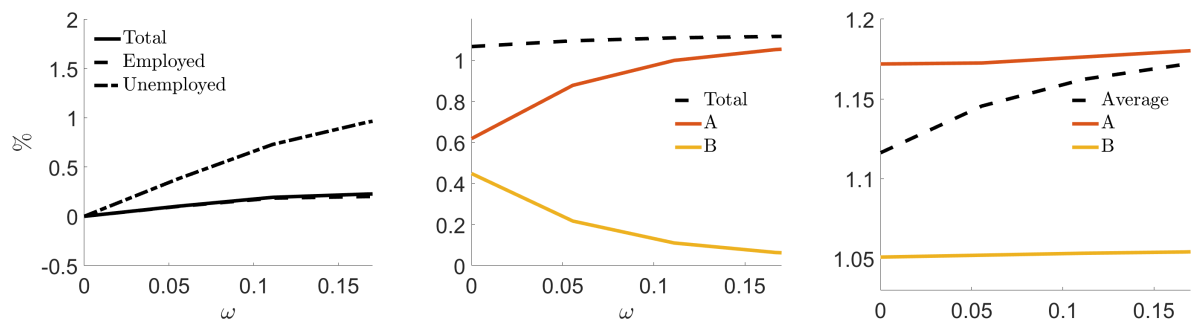
Figure D.6: Macro Outcomes for different unemployment benefits ϕ



(a) Welfare (b) Output (c) Wages

Note: Panel (a) shows welfare, aggregate and by employment status; Panel (b) shows output, total and by tier; and Panel (c) shows wages, average and by tier. All outcomes are equilibrium values under varying UI generosity ϕ.

Figure D.7: Macro outcomes for job creation subsidy ω



(a) Welfare (b) Output (c) Wages

Note: Panel (a) shows welfare, aggregate and by employment status; Panel (b) shows output, total and by tier; and Panel (c) shows wages, average and by tier. All outcomes are equilibrium values under varying job creation subsidy ω.